Fuzzy model assessment of the national life satisfaction index

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Introduction

Life satisfaction is one of the important indicators of life quality. Life satisfaction shows how people evaluate their life as a whole rather than their current feelings. It captures the reflective assessment of which life circumstances and conditions are important for subjective well-being. Numerous researches are dedicated to the problem of life satisfaction. There have been two major approaches to conceptualize life satisfaction: the so-called top-down and bottom-up approaches [4,6]. The top-down approach assumes that global satisfaction is a predispositional trait or personally which influences ones evaluation of satisfaction in various areas of life. The bottom-up approach, on the other hand, conceptualizes global satisfaction as being influenced by ones evaluation of satisfaction in various life domains. In studies of life satisfaction there are distinguished works based on the different national cultures [7,8] and countries [1, 3, 10], and also referring to different socio-demographic groups. [9, 11, 13]. Life satisfaction measurement problem was investigated in [12, 14], where mainly used structural equation models.

In this paper, we investigate national life satisfaction index for Azerbaijan. In order to do this we divide groups of households depending on income, into the following social groups:
- **1.Absolutely poor;**
- **2.Relatively poor;**
- **3.Low-income;**
- **4.Moderate means;**
- **5. Satisfied;**
- **6.Very satisfied.**

As domains of life satisfaction following areas were investigated:
- **work; income; house; health; leisure; natural environment.** In order to estimate National Life Satisfaction Index, instruments of fuzzy sets and fuzzy logic were used.

Formulation of model parameters

For development of fuzzy model we have used following linguistic variables [16] in the scope 0-1, mapped in triangular fuzzy number:

VD **Very dissatisfied**   **(0, 0, 0.25)**
DI **Dissatisfied**   **(0, 0.25, 0.5)**
MS **Averagely (Medium) satisfied**   **(0.25 ,0.5, 0.75)**
SA **Satisfied**   **(0.5, 0.75, 1)**
VS **Very satisfied**   **(0.75, 1, 1)**

In order to estimate level of life satisfaction of the social groups were used results of the investigation of the Center for Economic and Social Development of Azerbaijan, which is presented below.

In formulation of the social groups taken into account poverty line (116 AZN), average monthly wage (425 AZN) and social benefit (100 AZN), by government to the groups with low satisfaction level [2].

Taken into account level of income of population, were investigate next six social groups:

1. First group includes households, which have average monthly income of two times less than the poverty line, e.g. from 0 to 58 AZN. By UN classification, people, which have daily income of lower than 2 USD belong to **poor group**. But it does not reflect reality. If purchasing power of 1 USD in different countries is taken into account, this standard would not be same in different countries. In other words, in many African and Asian countries, 1 USD has more purchasing power than in our country. That is why average monthly wage of 58 AZN (73 USD) belongs to **absolutely poor**. For this reason, Azerbaijan government realizes targeted social assistance program. Aim of this program is to grant compensation by government equal to 100 AZN, for every person, which has income lower than poverty line. Considering first group of households, including group of population which cannot use this kind of social assistance. People from this group deprived a permanent place of residence, and are limited in use of healthcare services, do not enjoy the right to rest. On the other hand, they can be considered virtually unemployed. As if someone has an income of less than 58 AZN in a country where the average monthly wage is 425 AZN, then it means that this person has no job or, needs to be forced to work for a very low salary. Not having a job, these people are deprived of the opportunity to be included in the health insurance program. It means that government doesnt defense their health. Psychological position and material possibility of this group of people dont allow them a possibility to think about natural environment. People belonging to this group are far from getting a normal leisure services as prices in this sector in country are very high. So as they work without employment contract, they do not receive a coverage/allowance for vacation, and available income does not allow them to travel. As a result, we can underline that level of satisfaction with domains of life of this social group is **very dissatisfied (VD).** Weight (Wa) of this group in households 0,01, e.g 1 % .

2. Households with income of 58-116AZN, holding **poor** status because their average monthly income is 4-7 times less than the country average, belong to the second group. In other words status of the households belonging to the second group is lower than that of average household in Azerbaijan. This group can be divided into two sub-groups. First subgroup contains households having income less than critical poverty line (58-100 AZN), second subgroup contains households with average income of more than the critical poverty, but less than the living wage (100-116 AZN). Common feature of both groups is weak financial security. People from this group are primarily unemployed. They dont have opportunity to benefit from good health protection and rest programs, also they cant do anything about influence of environment on their health. Despite healthcare in Azerbaijan is partially free of charge, often people from this group cant have a psychologically healthy mode of life and to solve bigger health issues. Population of the households belonging to this group is considered to be not sufficiently provided in terms of housing. Since low income does not allow to have savings or benefit from mortgage programs they cant have house if not provided through inheritance or by other help means. They have to rent housing with the minimal facilities and therefore part of the income is spent for the rentals, which also contributes to the bad social state. These households are generally dont have good employment rates. Their income is not satisfactory so they are in constant search for better working opportunities. Current work is often a forced measure for them. Environmental conditions are not considered important for them. While looking for the rental housing main criteria is cost. Environment is one of the last factors for consideration. Bearing in mind all stated above, level of income satisfaction, level of environmental, housing, health and rest satisfaction can be considered as very dissatisfied **(VD)**, and the level of work satisfaction is dissatisfied **(DI)**. Weight of this group is 0,05.

3. **Third group** consists of people having income within the bounds of the living wage 116 - 232 AZN, or, in the best case twice as much. Although their income is greater than that of first two groups this group is still considered as **not sufficiently provided for**. This is due to the fact that their monthly income is 2-3 times less than the average wage level. Their housing conditions are not good, and households from third group often have rented housings, which also has impact on their power of consumption, and when rentals are considered main criteria is rental price again. They also do not consider environment as an important criteria while choosing place to live. People from the third group, in most of the cases, benefit from the free healthcare. Prices for private healthcare services are very high and third group representatives are not able to cover them. However they can afford that only in cases of extreme importance. When possible, they dont spend on the healthcare services. Spending on leisure is on the last place for the people belonging to third income group. Low income and high cost for leisure services limit their opportunities. They are using free of charge of very cheap leisure services like free parks and beaches which sometimes have ecological issues. People with low income very rarely use National Parks. They cant afford use of the touristic zones and hotels where daily cost is 50-110 AZN. All abovementioned gives us an opportunity to estimate level of satisfaction of this group as follows: job satisfaction **average/medium (MS)**, income, housing, healthcare, leisure, **dissatisfactory (DI)**. Weight of this group is 0,66.

4. People with average monthly income of 232-348 AZN can be regarded as **people with average income**. Teachers, medical workers, social sphere workers can be attributed to this group. In this group, population having average income is considered as employed, but is searching for better job opportunities. They have income source primarily form the countrys budget and are hoping for wages increase. When using healthcare services people from this group are primarily using state healthcare institutions, with the primary aim to spend less on healthcare. So they cannot afford use of more expensive private medical services. Financial expenses on the healthcare are mainly related to extreme situations. Possibilities for curing more serious health conditions are very limited. Using healthcare institutions abroad is not possible due to absence of savings. Average income level in Azerbaijan is low, and in reality it does not correspond to the standards of middle class.

Housing conditions are usually considered adequate, since income of people from this group allows having cheaper housing or social mortgage. Natural environment is of great importance for this group. They are trying to have a healthy lifestyle. However environmental conditions are not the absolute priority for them. Potential to have access to leisure is considered average. As most of the people from this group are working in the governmental sector they have official vacation and it is paid. Also they can benefit from the relatively cheap vacation programs supported by the trade unions, where they obtain different medical programs of an average quality. Considering what is said above, all domains of life satisfaction for this group can be estimated as **averagely/medium satisfied (MS)**. Weight of this group in total households is 0,22.

5. People from the fifth social group have monthly income of 348 500 AZN. They are employed having better qualities and positions, so their work satisfaction level is **satisfactory**. As compared to the other social groups it is necessary to underline better housing conditions, which is considered as **satisfactory**. Their income level allows use of any mortgage options or to rent housing with the better conditions. Health is their priority, so they use private clinics and can afford going abroad for medical services, when needed. Therefore they are satisfied with the medical services. People from that group have a healthier lifecycle and use only natural food products. **Moderately satisfied** with the environmental conditions. Weight of this group is 0,05.

6. Social group with income of 500AZN and more is **very satisfied** with their work, housing, healthcare and leisure opportunities, and are **satisfied** with natural environment. They efficiently use quality leisure opportunity, having vacations abroad, but also they are main users of leisure spots in country. Spending weekends and holidays in countryside. Weight of this group is 0,01. Analysis of the social groups and their respective satisfaction levels, carried out above, allows us to formalize fuzzy model of life satisfaction index, which is demonstrated in the Table 1.

Fuzzy model of life domains satisfaction   Tab. 1

Social groups	W^a	2013 116 AZN	Work	Income	House	Health	Leisure
1.Absolutely poor	0,01	0-58	VD (0, 0, 0.25)	VD (0, 0, 0.25)	VD (0, 0, 0.25)	VD (0, 0, 0.25)	VD (0, 0, 0.25)
2.Relatively poor	0,05	58-116	DI (0, 0.25, 0.5)	VD (0, 0, 0.25)	VD (0, 0, 0.25)	VD (0, 0, 0.25)	VD (0, 0, 0.25)
3.Low-income	0,66	116-232	MS (0.25,0.5,0.75)	DI (0, 0.25, 0.5)	DI (0, 0.25, 0.5)	DI (0, 0.25, 0.5)	DI (0, 0.25, 0.5)
4.Moderate means	0,22	232-348	MS (0.25,0.5,0.75)	MS (0.25,0.5,0.75)	MS (0.25,0.5,0.75)	MS (0.25,0.5,0.75)	MS (0.25,0.5,0.75)
5.Satisfactied	0,05	348-500	SA (0.5,0.75,1)	SA (0.5,0.75,1)	SA (0.5,0.75,1)	SA (0.5,0.75,1)	MS (0.25,0.5,0.75)
6.Very satisfactied	0,01	500 more than	VS (0.75,1,1)	VS (0.75,1,1)	VS (0.75,1,1)	VS (0.75,1,1)	SA (0.5,0.75,1)

Estimation of the national life satisfaction index

For estimation of the national life satisfaction index of the people we have developed fuzzy model, which is demonstrated in table 1. At the first stage weights of criteria (domains) of the life satisfaction were defined.

In order to do this, fuzzy analytic hierarchy process was used, which is proposed by P. Laarhoven and W. Pedrycz [15].

Following this methodology, fuzzy estimation of importance levels of human life domains is defined as:

1) Work absolutely important (ABI)=(7, 9, 9);
2) Income very important (VSI)=(5, 7,9);
3) House essentially important (ESI)=(3,5,7);
4) Health weakly important (WEI)=(1,3,5);
5) Leisure equally important (EQI)=(1,1,3);
6) Natural environment - equally important (EQI)=(1,1,3).

On the basis of parameters importance levels, pairwise comparison matrix was constructed:

$$
M = \begin{array}{c} \\ ABI \\ VSI \\ ESI \\ WEI \\ EQI \\ EQI \end{array}
\begin{array}{cccccc} ABI & VSI & ESI & WEI & EQI & EQI \end{array}
\left(\begin{array}{cccccc}
\frac{ABI}{ABI} & \frac{VSI}{ABI} & \frac{ESI}{ABI} & \frac{WEI}{ABI} & \frac{EQI}{ABI} & \frac{EQI}{ABI} \\
\frac{ABI}{VSI} & \frac{VSI}{VSI} & \frac{ESI}{VSI} & \frac{WEI}{VSI} & \frac{EQI}{VSI} & \frac{EQI}{VSI} \\
\frac{ABI}{ESI} & \frac{VSI}{ESI} & \frac{ESI}{ESI} & \frac{WEI}{ESI} & \frac{EQI}{ESI} & \frac{EQI}{ESI} \\
\frac{ABI}{WEI} & \frac{VSI}{WEI} & \frac{ESI}{WEI} & \frac{WEI}{WEI} & \frac{EQI}{WEI} & \frac{EQI}{WEI} \\
\frac{ABI}{EQI} & \frac{VSI}{EQI} & \frac{ESI}{EQI} & \frac{WEI}{EQI} & \frac{EQI}{EQI} & \frac{EQI}{EQI} \\
\frac{ABI}{EQI} & \frac{VSI}{EQI} & \frac{ESI}{EQI} & \frac{WEI}{EQI} & \frac{EQI}{EQI} & \frac{EQI}{EQI}
\end{array} \right)
= \begin{bmatrix}
\frac{(7,9,9)}{(9,9,7)} & \frac{(5,7,9)}{(9,9,7)} & \frac{(3,5,7)}{(9,9,7)} & \frac{(1,3,5)}{(9,9,7)} & \frac{(1,1,3)}{(9,9,7)} & \frac{(1,1,3)}{(9,9,7)} \\
\frac{(9,7,5)}{(9,9,7)} & \frac{(9,7,5)}{(5,7,9)} & \frac{(9,7,5)}{(3,5,7)} & \frac{(9,7,5)}{(1,3,5)} & \frac{(9,7,5)}{(1,1,3)} & \frac{(9,7,5)}{(1,1,3)} \\
\frac{(7,5,3)}{(9,9,7)} & \frac{(7,5,3)}{(5,7,9)} & \frac{(7,5,3)}{(3,5,7)} & \frac{(7,5,3)}{(1,3,5)} & \frac{(7,5,3)}{(1,1,3)} & \frac{(7,5,3)}{(1,1,3)} \\
\frac{(5,3,1)}{(9,9,7)} & \frac{(5,3,1)}{(5,7,9)} & \frac{(5,3,1)}{(3,5,7)} & \frac{(5,3,1)}{(1,3,5)} & \frac{(5,3,1)}{(1,1,3)} & \frac{(5,3,1)}{(1,1,3)} \\
\frac{(3,1,1)}{(9,9,7)} & \frac{(3,1,1)}{(5,7,9)} & \frac{(3,1,1)}{(3,5,7)} & \frac{(3,1,1)}{(1,3,5)} & \frac{(3,1,1)}{(1,1,3)} & \frac{(3,1,1)}{(1,1,3)} \\
\frac{(3,1,1)}{(3,1,1)} & \frac{(3,1,1)}{(3,1,1)} & \frac{(3,1,1)}{(3,1,1)} & \frac{(3,1,1)}{(3,1,1)} & \frac{(3,1,1)}{(3,1,1)} & \frac{(3,1,1)}{(3,1,1)}
\end{bmatrix}
$$

Then fuzzy weights of life domains were defined by using following formula, proposed by J Buckley [5]:

$$\tilde{r}_i = [\tilde{c}_{i1} \otimes \tilde{c}_{i2} \otimes ... \otimes \tilde{c}_{in}]^{\frac{1}{n}}, \qquad \forall i = 1, 2, ..., n, \qquad (1)$$

$$\tilde{W}_i = \frac{\tilde{r}_i}{\tilde{r}_1 \oplus ... \oplus \tilde{r}_n}, \qquad \forall i = 1, 2, ..., n, \qquad (2)$$

where $\tilde{c}_{ij}$ fuzzy element of pairwise comparison matrix, $\tilde{r}_i$ geometric mean column element of matrix C, $\otimes$ and $\oplus$ - multiply and addition operation for fuzzy number.

In our case r $\tilde{r}_i$ is calculated as:

$$\tilde{r}_1 = [(0.8 * 0.8 * 1.0 * 1.4 * 2.3 * 2.3), (1.0 * 1.3 * 1.8 * 3.0 * 9.0 * 9.0), (1.3 * 1.8 * 3.0 * 9.0 * 9.0 * 9.0)]^{\frac{1}{6}} =$$
$$[4.74, 568.62, 5117.58]^{\frac{1}{6}} = (1.29, 2.88, 4.16)$$

Other geometric means are equal to:

$$\tilde{r}_2 = (0, 92, 2.24, 4.155); \tilde{r}_3 = (0.55, 1.598, 3.23); \tilde{r}_4 = (0.184, 0.957, 2.31); \tilde{r}_5 = \tilde{r}_6 = (0.184, 0.32, 1.38);$$

Fuzzy sum geometric mean column elements of matrix C are:

$$\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3 \oplus \tilde{r}_4 \oplus \tilde{r}_5 \oplus \tilde{r}_6 = (3.31, 8.32, 16.61)$$

Then, on the basis of formula (2), we calculate fuzzy weights of life domains:

$\tilde{W}_1 = \frac{1.29, 2.88, 4.16}{16.61, 8.32, 3.31} = (0.078, 0.346, 1.25);$
$\tilde{W}_2 = \frac{0,92, 2.24, 4.155}{16.61, 8.32, 3.31} = (0.055, 0.269, 1.25);$
$\tilde{W}_3 = \frac{0.55, 1.598, 3.23}{16.61, 8.32, 3.31} = (0.033, 0.192, 0.975);$
$\tilde{W}_4 = \frac{0.184, 0.957, 2.31}{16.61, 8.32, 3.31} = (0.011, 0.115, 0.695);$
$\tilde{W}_5 = \frac{0.184, 0.32, 1.38}{16.61, 8.32, 3.31} = (0.011, 0.038, 0.417);$

After that, on the basis of obtained mean fuzzy weights, vector $W^c = (0.346, 0.269, 0.192, 0.115, 0.038, 0.038)$ is formulated.

On the second stage, index of satisfaction of social groups with life domains is calculated SQSI(i), for that matrix E (elements are compiled from the table 1) multiplied to the weights of domains (criteria) of vector W^c, e.g.

$$SQSI(i) = E \otimes W^c, (i = 1, ..., 6) \qquad (3)$$

where $\otimes$ - operation of conditional multiplication, according to every fuzzy number, required line of matrix E is multiplied to the corresponding element of the column of vector W^c.

So, in our case, index of satisfaction with the life domains of third social group is calculated as follows: =(0.09,0.35,0.6) corresponds to triangular fuzzy number which is close to dissatisfied (DI) level.

Result calculations of SQSI (i), (i=1,..,6), level of satisfaction by life domain of the social group are defined:

SQSI (1)= (0, 0, 0.25) VD;
SQSI (2)=(0, 0.09, 0.35) close to VD;
SQSI (3)= (0.09, 0.35, 0.60) close to DI;
SQSI (4)= (0.25, 0.50, 0.75) MS;
SQSI (5)= (0.5, 0.75, 1) SA;
SQSI (6)= (0.75, 1, 1) VS.

Results of calculated (fig.1) values of life satisfaction indices for the social groups show that values of indices of first, fourth, fifth and sixth groups fully coincide with the values of corresponding linguistic variables VD, MS, SA, VS. Values of the second social group index is close to VD and index of the third group is close to DI.

At the third stage, on the basis of values of social group indices, values of national life satisfaction index are defined. For this purpose, weight vector of the household Wa is multiplied to the matrix SQSIm, e.g.

where SQSIm- is a matrix, which is compiled from the elements of vector SQSI(i), (=1,2,,6). According to the Azerbaijan State Statistical Committee information [2] vector weights of households are as follows: Wa=(0.01, 0.05, 0.66, 0.22, 0.05, 0.01)

The analytical form of the process and results of the decision looks as follows:

$$
\text{NISLD} =(0.01, 0.05, 0.66, 0.22, 0.05, 0.01) * \begin{pmatrix} 0 & 0 & 0.25 \\ 0 & 0.09 & 0.35 \\ 0.09 & 0.35 & 0.60 \\ 0.25 & 0.50 & 0.75 \\ 0.5 & 0.75 & 1.00 \\ 0.75 & 1.00 & 1.00 \end{pmatrix} =
$$

=(0.01*0+0.05*0+0.66*0.09+0.22*0.25+0.05*0.5+0.01*0.75)+ (0.01*0+0.05*0.09+0.66*0.35+ +0.22*0.50+0.05*0.75+0.01*1) + (0.01*0.25+0.05*0.35+0.66*0.60+0.22*0.75+0.05*1.00+ +0.01*1.00)= (0+0+0.0594+0.055+0.025+0.0075)+(0+0.0045+0.231+0.11+0.0375+0.01)+ +(0.0025+0.0175+0.396+0.165+0.05+0.01)=(0.15;0.39;0.64)

Results of solution of the problem are demonstrated in figure 1.
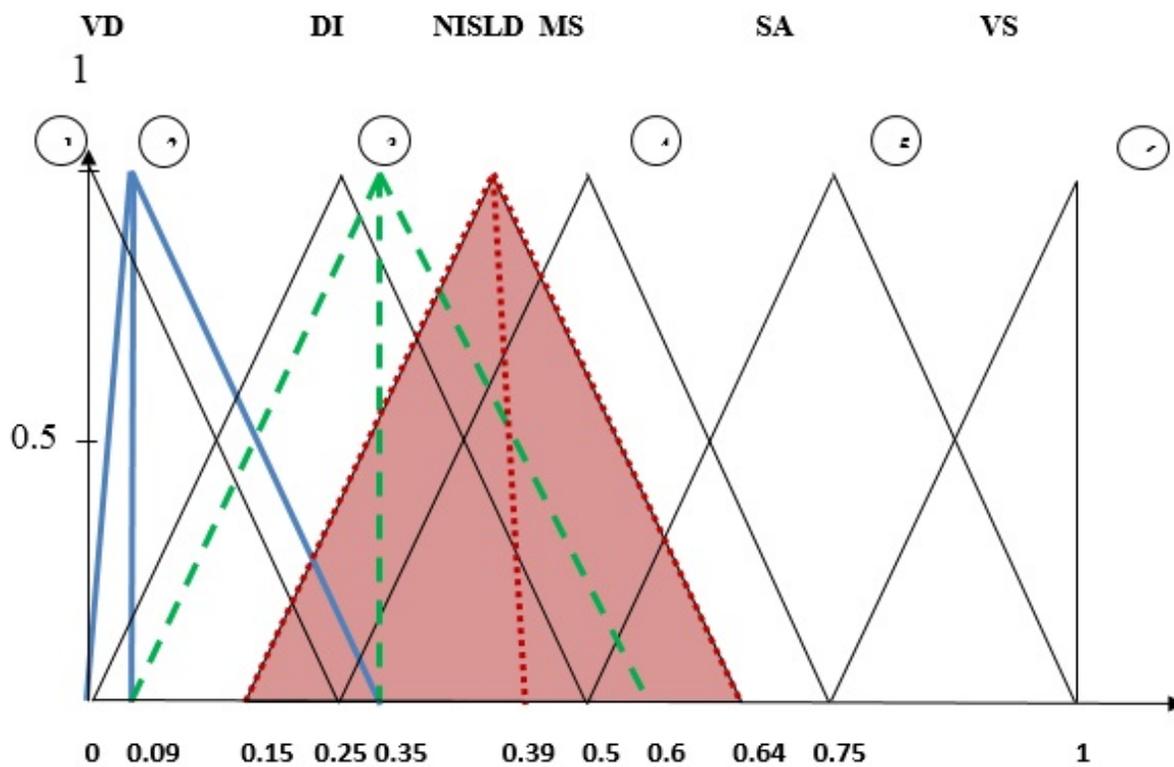
Fig.1 Results of problem solution

As it is shown on the figure 1, value of the national life satisfaction index (NISLD) of the Azerbaijan population was between values dissatisfied (DI) and moderately satisfied (MS) in 2013.

Conclusions

Application of instruments of fuzzy sets and fuzzy logic provides possibility to draw non-metric indicators of social system such as the level of satisfaction with life domains of social groups and national level of life satisfaction, in computational process. Results of investigation is reliable and adequate to the problem definition.

Research results obtained allow decision makers in the sphere of macro-socioeconomic system to correct parameters of governance. Furthermore in order to forecast and model this process it is necessary to explore present problem in the aspect of social groups and life spheres parameters modification probability.

References

[1] Alesina Alberto, Di Tella, R., MacCulloh, R. (2004). Inequality and Happiness: Are Europeans and Americans Different? Journal of Public Economics, 88 (9-10), 2009-2042.

[2] Azerbaijan State Statistical Committee. stat.gov.az. Retrieved from http://stat.gov.az.

[3] Blanchflower, D. G., Oswald, A. J. (2004). Well-Being Over Time in Britain and the USA. Journal of Public Economics, 88 (7-8), 1359-1386.

[4] Brief, A.P., Butcher, A.H., George, J.M., Link, K.E. (1993). Integrating Bottom-Up and Top-Down Theories of Subjective Well-Being: The Case of Health. Journal of Personality and Social Psychology, 64 (4),646-653.

[5] Buckley, J.J. (1985). Fuzzy hierarchical analysis. Fuzzy Sets and Systems, 17 (3), 233-247.

[6] Diener, E. (1984). Subjective well-being. Psychological Bulletin, 95, 542 575.

[7] Diener, E., Lucas, R.E. (2000). Explaining differences in societal levels of happiness: relatives standards need fulfillment, culture, and evaluation theory. Journal of Happiness Studies, 1, 41-78.

[8] Diener, E., Suh, E.M. (2000). Culture and subjective well-being. Cambridge, MA: MIT Press.

[9] Diener, E., Suh, E.M., Lucas, R.E., Smith, H.L. (1999). Subjective well-being: Three decades of progress. Psychological Bulletin, 125, 276-302.

[10] Di Tella, R., MacCulloh, R. J., Blanchflower, D. G. (2003). The Macroeconomics of Happiness. Review of Economics and Statistics. 85 (4), 809-827.

[11] Gilman, R.S, Huebner, E.S. (2000). Review of life satisfaction measures for adolescents. Behavior Change, 17, 178-195.

[12] Headey, B., Veenhoven , R., Wearing, A. (1991). Top-down versus bottom-up theories of subjective well-being. Social Indicators Research, 24, 81-100.

[13] Huebner, E.S. (1994). Preliminary development and validation of a multidimensional life satisfaction scale of children. Psychological Assessment, 6, 149-158.

[14] Lance, C.E., Lautenschlager, G.J., Sloan, C.E. Varca, P.E. (1989). A comparison between bottom-up, top-down and bidirectional models of relationship between global and life facet satisfaction. Journal of Personality, 57, 601-624.

[15] Laarhoven P.M., Pedrycz W.,(1983), A fuzzy extension of Saatys priority theory, Fuzzy Sets and Systems.- 11,(1-3).- 229-241

[16] Zadeh, L.A. (1975). The concept of linguistic variable and its application to approximate reasoning. Information Science, 8 (3), 199 249.